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MATHEMATICS QUESTION PAPER WITH SOLUTION

(CODE – 1ST SHIFT)

Address - Near Himfed Building, BCS Chowk, New Shimla (HP) - 171009
www.radianceiit.in | Phone: +91 8091157087 | E-mail: er.j.k.tiwari@gmail.com

Q.1 A line parallel to the straight line 2x-y=0 is tangent to the hyperbola $\frac{x^2}{4}\quad\frac{y^2}{2}\ 1$ at the point (x_1,y_1). Then $x_1^2\quad 5y_1^2$ is equal to :

(1) 6 (2) 10 (3) 8 (4) 5

Sol. **1**

$T : \frac{xx_1}{4} - \frac{yy_1}{2} = 1$ ....(1)

t : 2x — y = 0 is parallel to T

T : 2x — y = ...(2)

Now compare (1) & (2)

$$\frac{\frac{x_1}{4}}{2} = \frac{\frac{y_1}{2}}{1} = \frac{1}{\ }$$

$x_1=8/$ & $y_1 = 2/$

(x_1,y_1) lies on hyperbola $\frac{64}{4\ ^2} - \frac{4}{2\ ^2} = 1$

14= ^{2} Now

$= x_1^2 + 5y_1^2$

$$= \frac{64}{\ ^2} + 5\frac{4}{\ ^2}$$

$$= \frac{84}{14}$$

= 6 Ans.

Q.2 The domain of the function $f\ x\quad \sin\ ^{1}\ \frac{|x|\ 5}{x^2\quad 1}$ is (, a] [a,). Then a is equal to :

(1) $\frac{\sqrt{17}\ \ 1}{2}$ (2) $\frac{\sqrt{17}}{2}$ (3) $\frac{1\ \sqrt{17}}{2}$ (4) $\frac{\sqrt{17}}{2}\ 1$

Sol. **3**

$$-1 \leq \frac{|x|\ 5}{x^2\quad 1} \leq 1$$

$-x^2-1 \leq |x|+5 \leq x^2+1$

case - I

$-x^2-1 \leq |x|+5$

this inequality is always right x R

case - II

$|x|+5 \leq x^2+1$

$x^2 - |x| \geq 4$

$|x|^2-|x|-4 \geq 0$

$|x| \quad \frac{\sqrt{\ }\ 1\ \ 17}{2}\ |x| \quad \frac{1\sqrt{\ }17}{2} \geq 0$

$|x| \leq \frac{1\sqrt{17}}{2}$ $|x| > \frac{1\sqrt{17}}{2}$

not possible

$x \quad ,\frac{1\sqrt{17}}{2} \quad \frac{1\ \sqrt{17}}{2} \quad ,$

$a = \frac{1\sqrt{17}}{2}$

Q.3 If a function f(x) defined by $f\ x \quad \begin{matrix} ae^{x} \quad be^{x},\ 1\ \ x\ \ 1 \\ cx^{2} \qquad\qquad ,\ \ 1\ \ x\ \ 3 \\ ax^{2}\quad 2cx\ ,3\ \ x\ \ 4 \end{matrix}$ be continuous for some a, b,c R and f′(0)+f′(2) =e, then the value of a is :

(1) $\frac{1}{e^2\quad 3e\ \ 13}$ (2) $\frac{e}{e^2\quad 3e\ \ 13}$ (3) $\frac{e}{e^2\quad 3e\ \ 13}$ (4) $\frac{e}{e^2\quad 3e\ \ 13}$

Sol. **4**

f(x) is continuous

at x=1 $\boxed{ae \quad \frac{b}{\ } \quad c}$

at x=3 9c = 9a + 6c c=3a

Now f ′(0) +f ‵(2) = e

a − b + 4c = e

a − e (3a−ae) + 4.3a =

e a − 3ae + ae^2 + 12a

= e 13a − 3ae + ae^2=e

$a\ \boxed{\frac{e}{13\ \ 3e\ \ e^2}}$

Q.4 The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

(1) ,9 3, (2) 3, (3) ,9 (4) ,3 9,

Sol. **4**

$\frac{a}{r}.a.ar\ \ 27\ \ a\ \ 3$

$\frac{a}{r} + a + ar = S$

$\frac{1}{r} + 1 + r = \frac{S}{3}$

$r + \frac{1}{r} = \frac{S}{3} - 1$

$r + \frac{1}{r} \ge 2$ or $r + \frac{1}{r} \le -2$

$\frac{S}{3} \ge 3$ or $\frac{S}{3} \le -1$

$S \ge 9$ or $S \le -3$

$S \in (-\infty, -3] \cup [9, \infty)$

Q.5 If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

(1) $\{-1, 0, 1\}$ (2) $\{-2, -1, 1, 2\}$ (3) $\{0, 1\}$ (4) $\{-2, -1, 0, 1, 2\}$

Sol. **1**

$3y^2 \le 8 - x^2$

R : {(0,1), (0,−1), (1,0),(−1,0), (1,1), (1,-1)
(-1,1), (-1,-1), (2,0), (-2,0),(-2,0),(2,1),(2,-1), (-2,1),(-2,-1)}

R : {-2,-1,0,1,2} → {-1,0,-1}

Hence R^{-1} : {-1,0,1} → {-2,-1,0,1,2}

Q.6 The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :

(1) $-\frac{1}{2}(1 - i\sqrt{3})$ (2) $\frac{1}{2}(1 - i\sqrt{3})$ (3) $-\frac{1}{2}(\sqrt{3} - i)$ (4) $\frac{1}{2}(\sqrt{3} - i)$

Sol. **3**

$$\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$$

$$=\left(\frac{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)+i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)-i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}\right)^3$$

$$= \frac{\left(1 \cos \frac{5}{18} \, i \sin \frac{5}{18}\right)^3}{1 \cos \frac{5}{18} \, i \sin \frac{5}{18}}$$

$$= \frac{\left(2 \cos \frac{5}{36} \cos \frac{5}{36} \, i \sin \frac{5}{36}\right)^3}{2 \cos \frac{5}{36} \cos \frac{5}{36} \, i \sin \frac{5}{36}}$$

$$= \frac{36 \, \text{cis}\frac{5}{}^{\,3}}{\text{cis}\frac{5}{36}}$$

$$= \text{cis}\frac{5}{36}3 \quad \frac{5}{36}3$$

$$= \text{cis} \frac{10}{12}$$

$$= \text{cis}\frac{5}{6} = \boxed{\frac{\sqrt{3}}{2} \quad \frac{i}{2}}$$

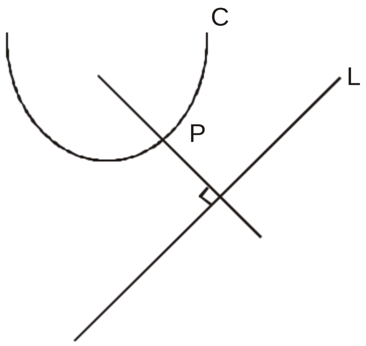
Q.7 Let P(h,k) be a point on the curve $y=x^2+7x+2$, nearest to the line, $y=3x-3$. Then the equation of the normal to the curve at P is:

(1) $x+3y-62=0$ (2) $x-3y-11=0$ (3) $x-3y+22=0$ (4) $x+3y+26=0$

Sol. **4**

C : $y = x^2 + 7x+2$

Let P : (h, k) lies on



Curve = $k = h^2+7h+2$

Now for shortest distance

$M_T\,|_P^C = m_L = 2h+7 = 3$
h =-2
k=-8
P : (-2,-8)
equation of normal to the curve is perpendicular to L : 3x − y = 3
N : x + 3y =
Pass (-2,-8)
= -26
N : x + 3y + 26 = 0

Q.8 Let A be a 2×2 real matrix with entries from 0,1 and | A | 0. Consider the following two statements:
(P) If A I_2, then | A | 1
(Q) If |A|=1, then tr(A) =2,
where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then:
(1) Both (P) and (Q) are false
(2) (P) is true and (Q) is false
(3) Both (P) and (Q) are true
(4) (P) is false and (Q) is true

Sol. **4**

P:A= $\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}$ I_2 & |A| 0 & |A| = 1(false)

Q:A= $\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}$ = 1 then Tr(A) = 2 (true)

Q.9 Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

(1) $\frac{4}{17}$ (2) $\frac{8}{17}$ (3) $\frac{2}{5}$ (4) $\frac{2}{3}$

Sol. **2**

$\boxed{1\text{to } 30}$
boxI
Prime on I
{2,3,5,7,11,13,17,19,23,29}
$\boxed{31\text{to } 50}$
box II
Prime on II
{31,37,41,43,47}
A : selected number on card is non - prime
P(A) = P(I).P(A/I) + P(II). P(A/II)

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} . \frac{15}{20}$$

Now, $P(I/A) = \dfrac{P(II).P(A/I)}{P(A)}$

$$\frac{\frac{1}{2}.\frac{20}{30}}{\frac{1}{2}.\frac{20}{30} \quad \frac{1}{2}.\frac{15}{20}} \quad \frac{\frac{2}{3}}{\frac{2}{3} \quad \frac{3}{4}} = \frac{8}{17}$$

Q.10 If p(x) be a polynomial of degree three that has a local maximum value 8 at x=1 and a local minimum value 4 at x=2; then p(0) is equal to :

(1) 12 (2) -12 (3) -24 (4) 6

Sol. **2**

$p'(1) = 0$ & $p'(2) = 0$

$p'(x) = a(x-1)(x-2)$

$p(x) = a \quad \frac{x^3}{3} \quad \frac{3x^2}{2} 2x \quad b$

$p(1) = 8 \quad a \quad \frac{1}{3} \quad \frac{3}{2} 2 \quad +b = 8$ ...(i)

$p(2) = 4 \quad a \quad \frac{8}{3} \quad \frac{3.4}{2} 2.2 + b = 4$ ......(ii)

from equation (i) and (ii)

a = 24 & b = −12

$p(0) = b = \boxed{12}$

Q.11 The contrapositive of the statement "If I reach the station in time, then I will catch the train'' is:

(1) If I will catch the train, then I reach the station in time.
(2) If I do not reach the station in time, then I will catch the train.
(3) If I do not reach the station in time, then I will not catch the train.
(4) If I will not catch the train, then I do not reach the station in time.

Sol. **4**

Statement p and q are true
Statement, then the contra positive of the implication
p q = (~q) (~p)
hence correct Ans. is 4

Q.12 Let and be the roots of the equation, $5x^2+6x-2=0$. If S_n ^{n} $_n$, n=1,2,3,....., then:

(1) $5S_6+6S_5+2S_4=0$ (2) $6S_6+5S_5=2S_4$
(3) $6S_6+5S_5+2S_4=0$ (4) $5S_6+6S_5=2S_4$

Sol. **4**

$5x^2 + 6x - 2 = 0 \quad = 5\ ^2 + 6\ = 2$

Simillarly

$6\ -2=-5\ ^2$

$S_6=\ ^6+\ ^6$

$S_5=\ ^5+\ ^5$

$S_4=\ ^4+\ ^4$

Now $6S_5 - 2S_4$

$= 6\ ^5-2\ ^4+6\ ^5-2\ ^4$

$= a^4(6\ -2) + \ ^4(6\ -2)$

$= \ ^4(-5\ ^2) + \ ^4(-5\ )$

$= -5(\ ^6+\ ^6)$

$= -5S_6$

$= 6S_5 + 5S_6 = 2S_4$

Q.13 If the tangent to the curve y=x+siny at a point (a,b) is parallel to the line joining and $0,\frac{3}{2}$ $\frac{1}{2},2$, then:

(1) $b\ \frac{}{2}\ a$ (2) |a+b|=1 (3) |b-a|=1 (4) b=a

Sol. **3**

$\left.\frac{dy}{dx}\right|_{p(a,b)}^{c} = \frac{2\ \frac{3}{2}}{\frac{1}{2}\ 0}$

1 cosb | 1 p : (a,b)lies on curve

cosb 0 | b a sin b

$\boxed{b\ a\ 1}$

b a 1

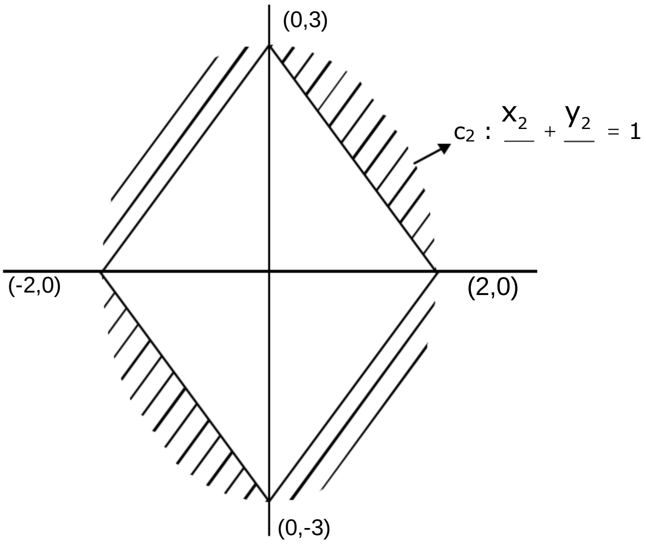
$\boxed{|\ b\ a|\ 1}$

Q.14 Area (in sq. units) of the region outside $\frac{|x|}{2}\ \frac{|y|}{3}\ 1$ and inside the ellipse $\frac{x^2}{4}\ \frac{y^2}{9}\ 1$ is:

(1)3 2 (2)6 2 (3)64 (4)34

Sol. **2**

$c_1 : \frac{|x|}{2} + \frac{|y|}{3} = 1$



$A = 4\frac{ab}{4} \quad \frac{1}{2}.2.3$

$A = \ .2.3 - 12$

$A = 6(\ -2)$

Q.15 If $|x|<1, |y|<1$ and $x \ y$, then the sum to infinity of the following series $(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+....$ is:

(1) $\frac{x \ y \ xy}{1 \ x \ 1 \ y}$ (2) $\frac{x \ y \ xy}{1 \ x \ 1 \ y}$ (3) $\frac{x \ y \ xy}{1 \ x \ 1 \ y}$ (4) $\frac{x \ y \ xy}{1 \ x \ 1 \ y}$

Sol. **2**

$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+....$

$= \frac{1}{(x \ y)} x^2 \ y^2 \quad x^3 \ y^3 \quad x^4 \ y^4 \quad ...$

$= \frac{\frac{x^2}{1 \ x} \quad \frac{y^2}{1 \ y}}{x \ y}$

$= \frac{x^2(1 \ y) \ y^2(1 \ x)}{(1 \ x)(1 \ y)(x \ y)}$

$= \frac{(x^2 \ y^2) \ xy(x \ y)}{(1 \ x)(1 \ y)(x \ y)} = \frac{(x \ y) \ xy(x \ y)}{(1 \ x)(1 \ y)(x \ y)}$

$= \frac{x \ y \ xy}{\ }$

Q.16 Let 0, 0 be such that ^{3} ^{2} 4 . If the maximum value of the term independent of x in the binomial expansion of $x^{\frac{1}{9}}$ $x^{-\frac{1}{6}}$ ^{10} is 10k, then k is equal to:

(1) 176 (2) 336 (3) 352 (4) 84

Sol. 2

For term independent of x

$T_{r+1} = {}^{10}C_{r} \, x^{\frac{1}{9}} {}^{10-r} \, x^{-\frac{1}{6}} {}^{r}$

$T_{r+1} = {}^{10}C_{r} {}^{10-r} {}^{r} . x^{\frac{10-r}{9}} . x^{-\frac{r}{6}}$

$\frac{10-r}{9} - \frac{r}{6} = 0 \quad r=4$

$T_5 = {}^{10}C_r \; {}^{6} . {}^{4}$

AM GM

Now $\frac{\frac{3}{2} \frac{3}{2} \frac{2}{2} \frac{2}{2}}{4} \geq \sqrt[4]{\frac{6 \cdot 4}{2^4}}$

$\frac{4}{4} \geq \frac{6 \, 4}{2^4}$

$6 . 4 \leq 2^4$

${}^{10}C_4 . 6 . 4 \leq {}^{10}C_4 \, 2^4$

$T_5 \leq {}^{10}C_4 2^4$

$T_5 \leq \frac{10!}{6!4!} . 2^4$

$T \leq \frac{10.9.8.7.2^4}{}$

maximum value of T_5 = 10. 3.7. 16 = 10k

k = 16.7.3

k = 336

Q.17 Let S be the set of all R for which the system of linear equations

2x-y+2z=2

x-2y+ z=-4

x+ y+z=4

has no solution. Then the set S

(1) is an empty set. (2) is a singleton.

(3) contains more than two elements. (4) contains exactly two elements.

Sol. 4

For no solution

=0& $_1$|$_2$|$_3$ 0

$$= \begin{vmatrix} 2 & 1 & 2 \\ 1 & 2 & \\ 1 & & 1 \end{vmatrix} 0$$

$2(-2-\ ^2)+1(1-\ )+2(\ +2)=0$

$-4-2\ ^2+1-\quad +2\ +4=0$

$-2\ ^2+\ +1=0$

$2\ ^2-\quad -1=0 \qquad =1,-1/2$

Equation has exactly 2 solution

Q.18 Let X x N : 1 x 17 and Y= ax b : x X and a,b R, a 0 . If mean and variance of elements of Y are 17 and 216 respectively then a+b is equal to:

(1)-27 (2) 7 (3)-7 (4) 9

Sol. 3

X : {1,2,.....17}

Y : {ax+b : x X & a, b R , a>o}

Given Var(Y) = 216

$\dfrac{y_1{}^2}{n} - (\text{mean})^2 = 216$

$\dfrac{y_1{}^2}{17} - 289 = 216$

$y_1 \quad 8585$

$(a+b)^2 + (2a+b)^2 + \ldots + (17a+b)^2 = 8585$

$105a^2 + b^2 + 18ab = 505$ ....(1)

Now $y_1 = 17 \times 17$

$a(17\times9) + 17.b = 17 \times 17$

$9a + b = 17$ ....(2)

from equation (1) & (2)

a = 3 & b = -10

a+b = −7

Q.19 Let y=y(x) be the solution of the differential equation, $\frac{2\ \sin x}{y\ \ 1}\cdot\frac{dy}{dx}$ cos x,y 0,y0 1. If

$y \quad a$, and $\frac{dy}{dx}$ at x is b, then the ordered pair (a,b) is equal to:

(1) $2,-\frac{3}{2}$ (2) (1,1) (3) (2,1) (4) (1,-1)

Sol. **2**

$\frac{dy}{y\ \ 1} \quad \frac{\cos x\, dx}{2\ \ \sin x}$

ln |y+1| = -ln |2+sin x| + k

(0,1)

k = ln 4

Now C : (y+1) (2+sin x) = 4

y( )=a (a+1) (2+0)=4 (a=1)

$\left.\frac{dy}{dx}\right|_{x} \qquad b \quad b1\frac{2\ \ 0}{1\ \ 1}$

b = 1

(a,b) = (1,1)

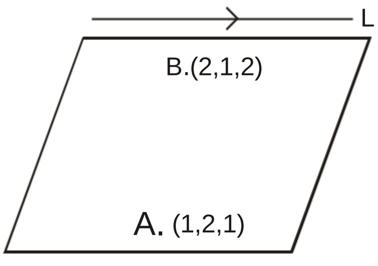
Q.20 The plane passing through the points (1,2,1), (2,1,2) and parallel to the line, 2x=3y, z=1 also passes through the point:

(1) (0,-6,2) (2) (0,6,-2) (3) (-2,0,1) (4) (2,0,-1)

Sol. 3

L : $\begin{matrix} 2x \quad 3y \\ z \quad 1 \end{matrix}$ $\begin{matrix} P:(0,0,1) \\ Q:(3,2,1) \end{matrix}$

V_L Dr of line (3,2,0)



$n_p = AB \times V_L$

$n_p = \langle 1, 1, 1\rangle \times \langle 3, 2, 0\rangle$

$n_p = \langle 2, 3, 5\rangle$

Plane : −2(x-1)+3(y-2)+5(z -1)=0

Plane : -2x+3y+5z+2–6-5=0
Plane : 2x – 3y - 5z = -9

Q.21 The number of integral values of k for which the line, 3x+4y=k intersects the circle, $x^2+y^2-2x-4y+4=0$ at two distinct points is...............

Sol. **9**

c : (1,2) & r = 1

|cp| < r

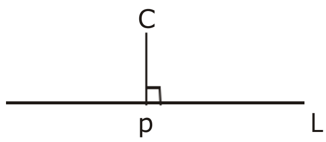
$$\left|\frac{3.1\ \ 4.2\ \ k}{5}\right|\ \ 1$$

|11-k|<5

–5<k-11<5

6<k<16

k = 7, 8, 9, ..., 15 total 9 value of k



Q.22 Let a,b and c be three unit vectors such that $|a\ \ b|^2\ \ |a\ \ c|^2\ 8$. Then $|a\ \ 2b|^2\ \ |a\ \ 2c|^2$ is equal to :

Sol. **2**

$$|a\ \ b|^2\ \ |a\ \ c|^2\ \ 8$$

a b . a b + a c a c =8

$a^2+b^2 - 2a.b + a^2+c^2-2a.c=8$

$2a^2+b^2+c^2-2a.b -2a.c=8$

a.b +a.c=-2

Now $|\ a\ 2b\ |^2 + |\ a\ \ 2c\ |^2$

$= 2a^2+4b^2+4c^1+4\bar{a}.\bar{b}+4\bar{a}.\bar{c}$

=2+4+4+4(–2)

= 2

Q.23 If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is.........

Sol. **309**

EHMORT

E-----	= 5!
H-----	= 5!
ME----	= 4!
MH----	= 4!
MOE---	= 3!
MOH---	= 3!
M O R - - -	= 3!
MOTE--	= 2!
MOTHER	= 1
	309

Q.24. If $\lim_{x \to 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} = 820$, $n \in N$ then the value of n is equal to :

Sol. 40

$$\lim_{x \to 1} \frac{x - 1}{x - 1} + \frac{x^2 - 1}{x - 1} + \dots + \frac{x^n - 1}{x - 1} = 820$$

$1 + 2 + 3 + \dots\dots + n = 820$

$\Sigma n = 820$

$$\frac{n(n+1)}{2} = 820$$

$n = 40$

Q.25 The integral $\int_0^2 ||x - 1| - x|\, dx$ is equal to :

Sol. 1.5

$$\int_0^2 ||x - 1| - x| dx$$

$$= \int_0^1 |1 - x - x|\, dx + \int_1^2 |x - 1 - x|\, dx$$

$$= \int_0^1 |2x - 1|\, dx + \int_1^2 1\, dx$$

$$= \int_0^{\frac{1}{2}} (1 - 2x)\, dx + \int_{\frac{1}{2}}^1 (2x - 1)\, dx + \int_1^2 1 dx$$

$$= \left(\frac{1}{2} - 0\right) - \left(\frac{1}{4} - 0\right) + \left(1 - \frac{1}{4}\right) - \left(1 - \frac{1}{2}\right) + 1$$

$$= \frac{1}{2} - \frac{1}{4} + \frac{3}{4} - \frac{1}{2} + 1$$

$$= \frac{3}{2}$$