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**MATHEMATICS QUESTION PAPER WITH
SOLUTION
(CODE – 2ND SHIFT)**



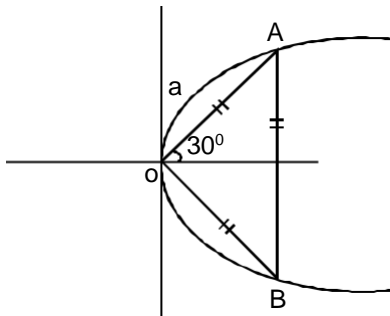
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Address - Near Himfed Building, BCS Chowk, New Shimla (HP) - 171009
www.radianceit.in | Phone: +91 8091157087 | E-mail: er.j.k.tiwari@gmail.com

Q.5 The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is:

- (1) $128\sqrt{3}$ (2) $192\sqrt{3}$ (3) $64\sqrt{3}$ (4) $256\sqrt{3}$

Sol. (2)



A : $(a \cos 30^\circ, a \sin 30^\circ)$ lies on parabola

$$\frac{a^2}{4} = 8 \cdot \frac{a \cdot \sqrt{3}}{2}$$

$$a = 16\sqrt{3}$$

$$\text{Area of equilateral} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \cdot 16 \cdot 16 \cdot 3$$

$$= 192 \sqrt{3}$$

Q.6 The imaginary part of $3 - 2\sqrt{54}^{1/2} - 3 + 2\sqrt{54}^{1/2}$ can be :

- (1) $\sqrt{6}$ (2) $2\sqrt{6}$ (3) 6 (4) $\sqrt{6}$

Sol. (2)

$$3 - 2i\sqrt{54}^{1/2} - 3 + 2i\sqrt{54}^{1/2}$$

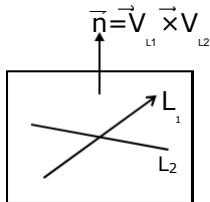
$$= 9 - 6i^2 - 2 \cdot 3i\sqrt{54}^{1/2} + 9 - 6i^2 + 2 \cdot 3i\sqrt{54}^{1/2}$$

$$= 3\sqrt{6i}^{21/2} - 3\sqrt{6i}^{21/2}$$

$$= 3\sqrt{6i} - 3\sqrt{6i} = -2\sqrt{6i}$$

Q.7 A plane passing through the point (3,1,1) contains two lines whose direction ratios are 1, -2, 2 and 2, 3, -1 respectively. If this plane also passes through the point (4, -3, 5), then k is equal to:
 (1) -5 (2) 10 (3) 5 (4) -10

Sol. (3)



$n_p = (4, 5, 7)$

Equation of plane :

$P : -4(x-3) + 5(y-1) + 7(z-1) = 0$

$P : -4x + 5y + 7z + 12 - 5 - 7 = 0$

$P : 4x - 5y - 7z = 0$

Pass (4, -3, 5)

$4 + 15 - 35 = 0$

$4 = 20$

$= 5$

Q.8 Let $A = \{X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1\}$, where $P = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \end{pmatrix}$, then the set A:

- (1) contains more than two elements
- (3) contains exactly two elements

- (2) is a singleton.
- (4) is an empty set.

Sol. (3)

x

$x = y \text{ \& } x^2 + y^2 + z^2 = 1$

z

$PX = 0$

$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

$\begin{pmatrix} 1 & 9 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

$x + 2y + z = 0 \dots\dots (1)$

$-2x + 3y - 4z = 0 \dots (2)$

$x + 9y - z = 0 \dots\dots\dots (3)$

from (1) & (3)

$2x + 11y = 0$

from (1) & (2)

$2x + 11y = 0$

from (2) & (3)

$$-6x - 33y = 0$$

$$2x + 11y = 0$$

0 put in (1)

$$-7y + 2z = 0$$

$$\text{Now } \frac{11y^2}{2} + y^2 + \frac{7y^2}{2} = 1$$

$$y^2(121 + 1 + 49) = 4$$

$$y^2(171) = 4$$

$$y = \pm \frac{2}{\sqrt{171}} \quad x = \frac{7}{\sqrt{171}} \quad z = \frac{11}{\sqrt{171}} \quad \text{Only two pair possible}$$

- Q.9** The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x=0$ is:
 (1) $y+4x=2$ (2) $2y+x=4$ (3) $x+4y=8$ (4) $y=4x+2$

Sol. (3)

$$\text{at } x = 0 \quad y = 1 + \cos^2(0) = 2$$

$$p : (0, 2)$$

$$\text{Now } y^1 = (1+x)^{2y} \quad \frac{2y}{1+x} \ln(1+x) \cdot 2y \sin 2(\sin^{-1}x) \cdot \frac{1}{1+x}$$

$$y \Big|_{(0,2)} = 4 - 0$$

$$N_o : y - 2 = \frac{1}{4} (x-0)$$

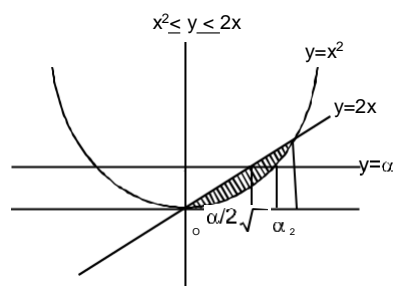
$$N_o : 4y - 8 = -x$$

$$N_o : \boxed{x + 4y - 8 = 0}$$

- Q.10** Consider a region $R = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 2x\}$. If a line y divides the area of region R into two equal parts, then which of the following is true?

- (1) $3^3 - 6^2 = 160$ (2) $3^2 - 8^{3/2} = 80$
 (3) $3^3 - 6^{3/2} = 16$ (4) $3^2 - 8 = 80$

Sol. (2)



$$A = \text{Area} = \int_0^2 (2x - x^2) dx = 4 - \frac{8}{3} = \frac{4}{3}$$

Now $\int_0^2 (2x - x^2) dx + \int_2^3 (x^2) dx = 2A$

$$\frac{2}{4} - \frac{2^3}{24} + \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3} - \frac{3^3}{24} = \frac{4}{6}$$

$$\frac{2}{4} + \frac{\sqrt{2}}{2} - \frac{2^2}{3} - \frac{\sqrt{3}}{3} = \frac{4}{6}$$

$$-3^2 + 8\sqrt{2} - 8 = 0$$

$$3^2 - 8\sqrt{2} + 8 = 0$$

1

Q.11 Let $f : (-1, 1) \rightarrow \mathbb{R}$ be defined by $f(0)=1$ and $f'(x) = -\log_e(1-x), x \in (-1, 1)$. Then the function f :

- (1) increases in $(-1, 1)$
- (2) decreases in $(-1, 0)$ and increases in $(0, 1)$
- (3) increases in $(-1, 0)$ and decreases in $(0, 1)$
- (4) decreases in $(-1, 1)$.

Sol. (4)

$$f(x) = \int -\log_e(1-x) dx$$

$$f' = \frac{x \cdot \frac{1}{1-x} \ln(1-x)}{x^2}$$

$$f = \frac{1 - \frac{1}{1-x} \ln(1-x)}{x^2}$$

$$f' > 0 \quad x \in (-1, 1)$$

Q.12 Which of the following is a tautology?

- (1) $(p \vee q) \vee (q \vee p)$
- (2) $(\sim p) \vee (p \wedge q) \vee q$
- (3) $(q \vee p) \vee (\sim p \wedge q)$
- (4) $(\sim q) \vee (p \wedge q) \vee q$

Sol. (2)

$\sim p$	$p \vee q$	$\sim p \vee (p \wedge q)$	$\sim p \vee (p \wedge q) \vee q$
F	T	F	T
F	T	F	T
T	T	T	T
T	F	F	T

Q.13 Let $f(x)$ be a quadratic polynomial such that $f(-1)+f(2)=0$. If one of the roots of $f(x)=0$ is 3, then its other roots lies in:

- Sol.** (1) (0,1) (2) (1,3) (3) (-1,0) (4) (-3,-1)

(3)

$$\text{Let } f(x) = a(x-3)(x-)$$

$$f(-1)+f(2)=0$$

$$a[(-1-3)(-1-)+(2-3)(2-)] = 0$$

$$a[4+4-2+]=0$$

$$5+2=0$$

$$\boxed{\frac{2}{5}}$$

Q.14 Let S be the sum of the first 9 terms of the series :

$$\{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\} + \{x^4+(k+6)a\} + \dots \quad \text{where } a > 0 \text{ and } a < 1.$$

If $S = \frac{x^{10} - x - 45a(x-1)}{x-1}$, then k is equal to:

- (1) 3 (2) -3 (3) 1 (4) -5

Sol. (2)

$$S = \{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\} \text{ up to 9 term}$$

$$S = (x+x^2+\dots+x^9) + a\{k+(k+2)+(k+4)+\dots\text{up to 9 term}\}$$

$$S = \frac{x(1-x^9)}{1-x} + a\{9k+2.36\}$$

$$S = \frac{x^{10}-x}{x-1} + 9ak + 72a$$

$$S = \frac{x^{10}-x-45a(x-1)}{x-1} = \frac{x^{10}-x-(9k+72)a(x-1)}{x-1}$$

$$= 45 = 9k + 72$$

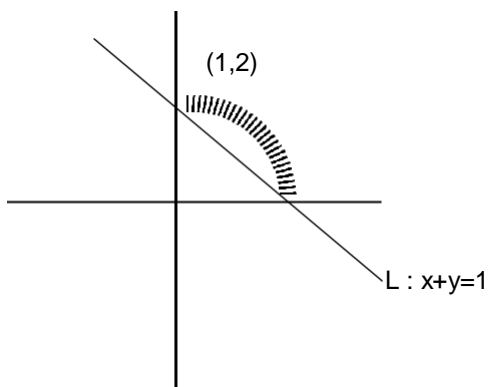
$$9k = -27$$

$$k = -3$$

Q.15 The set of all possible values of x in the interval $(0, 1)$ for which the points $(1,2)$ and $(\sin x, \cos x)$ lie on the same side of the line $x+y=1$ is:

- (1) $0, \frac{3}{4}$ (2) $0, \frac{1}{2}$ (3) $0, \frac{3}{4}$ (4) $\frac{1}{4}, \frac{3}{4}$

Sol. (2)

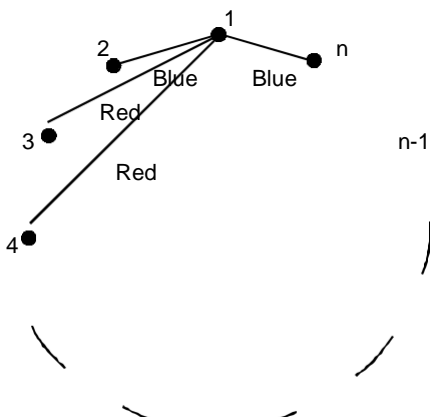


(sin, cos) lie on $x^2 + y^2 = 1$
 Shaded points satisfy
 $(0, \frac{1}{2})$

Q.16 Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is:

- (1) 201 (2) 199 (3) 101 (4) 200

Sol. (1)



Red line = 99 blue line

$${}^n C_2 - n = 99n$$

$$n(n-1) = 100n$$

$$n-1 = 100$$

$$n = 101$$

Q.17 If a curve $y=f(x)$, passing through the point $(1,2)$ is the solution of the differential equation, $2x^2dy=(2xy+y^2)dx$, then $\frac{1}{f'(2)}$ is equal to:

- (1) $\frac{1}{1 + \log_e 2}$ (2) $1 + \log_e 2$ (3) $\frac{1}{1 - \log_e 2}$ (4) $\frac{1}{1 - \log_e 2}$

Sol. (3)

$$2 \frac{dy}{dx} = 2 \frac{y}{x} + \frac{y^2}{x} \text{ -HDE}$$

$$y = vx$$

$$2v \cdot x \frac{dv}{dx} = 2v + v^2$$

$$2 \frac{dv}{v^2} = \frac{dx}{x}$$

$$-\frac{2}{v} = \ln x + c$$

$$\frac{2x}{y} = \ln x + c$$

$$(1,2)$$

$$c = -1$$

$$c : \ln x - \frac{2x}{y} = 1$$

$$\text{For } f(1/2) \quad \ln \frac{1}{2} + \frac{2}{2y} = 1$$

$$y = \frac{1}{1 - \ln 2}$$

Q.18 For some $0 < e < 1$, if the eccentricity of the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 5 times the \sqrt{e}

eccentricity of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the length of the latus rectum of the ellipse, is:

- (1) $\frac{4\sqrt{5}}{3}$ (2) $\frac{2\sqrt{5}}{3}$ (3) $2\sqrt{6}$ (4) $\sqrt{30}$

Sol. (1)

$$H : x^2 - y^2 \sec^2 = 10$$

$$E : x^2 \sec^2 + y^2 = 5$$

$$\sqrt{1 - \frac{10 \cos^2}{10}} = \sqrt{5} \sqrt{1 - \frac{5 \cos^2}{5}}$$

$$1 + \cos^2 = 5 - 5 \cos^2$$

$$6 \cos^2 = 4$$

$$\cos = \sqrt{\frac{2}{3}}$$

$$l(\text{LR}) \text{ of ellipse} = \frac{2.5 \cos^2}{\sqrt{5}}$$

$$= 2\sqrt{5} \cdot \frac{2}{3} = \frac{4\sqrt{5}}{3}$$

Q.19 $\lim_{x \rightarrow 0} \tan^{-x} \frac{1}{4}$ is equal to:

(1) e

(2) e^2

(3) 2

(4) 1

Sol. (2)

$$\lim_{x \rightarrow 0} \tan^{-x} \frac{1}{4} = e^L$$

$$L = \lim_{x \rightarrow 0} \frac{\tan^{-x} \frac{1}{4} - 1}{x}$$

$$L = \lim_{x \rightarrow 0} \frac{\frac{1}{\tan^x \frac{1}{4}} - 1}{x}$$

$$L = \lim_{x \rightarrow 0} \frac{\tan^x \frac{1}{4} - 1}{x \tan^x \frac{1}{4}}$$

$$L = +2$$

$$\text{Ans. } e^2$$

Q.20 Let $a, b, c \in \mathbb{R}$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$$A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$$A^T A = I$$

$$\text{satisfies } A^T A = I, \text{ then a value of } abc \text{ can be:}$$

$$(1) \frac{2}{3}$$

$$(2) 3$$

$$(3) \frac{1}{3}$$

$$(4) \frac{1}{3}$$

Sol. (4)

$$a^3 + b^3 + c^3 = 2$$

$$A^T A = I$$

$$\begin{matrix} a & b & c & & a & b & c & & 1 & 0 & 0 \end{matrix}$$

$$\begin{matrix} b & c & a & & b & c & a & = & 0 & & 1 & 0 \\ c & a & b & & c & a & b & & 0 & & 0 & 1 \end{matrix}$$

$$= a^2 + b^2 + c^2 = 1$$

$$\& ab + bc + ca = 0$$

$$\text{Now } (a+b+c)^2 = a^2 + 2ab + b^2 + 2bc + c^2 + 2ca$$

$$a^2 = 1 + 0 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

$$\text{Now } a^3 - 3abc = a(a^2 - 3bc)$$

$$2 - 3abc = \pm 1(1 - 0)$$

$$2 - 3abc = \pm 1$$

$$\begin{array}{l} \begin{array}{c} (+) \quad \quad \quad (-) \\ \diagdown \quad \quad \quad \diagup \\ 3abc = 1 \quad \quad 3abc = 3 \\ \boxed{abc = -1} \quad \quad \boxed{abc = 1} \end{array} \end{array}$$

Q.21 Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$. If O is the origin and

$$OB \cdot OP = 3 \left| OA \cdot OP \right|^2, \text{ then } \lambda \text{ is equal to } \underline{\quad}$$

Sol. 0.8

$$OA = \langle 1, 1, 1 \rangle, OB = \langle 2, 1, 3 \rangle$$

$$\frac{\lambda}{A} : \frac{1}{B}$$

$$OP = \frac{2\lambda}{1} \hat{i} + \frac{1}{1} \hat{j} + \frac{3\lambda}{1} \hat{k}$$

$$OB \cdot OP = \frac{2(2\lambda)}{1} + 1 + \frac{3(3\lambda)}{1}$$

$$= \frac{14\lambda + 6}{1}$$

$$|OA - OP|^2 = |OA|^2 + |OP|^2 - 2 \cdot OA \cdot OP$$

$$3. \frac{(2-1)^2 + (-1)^2 + (3-1)^2}{(1)^2} - \frac{2 \cdot 1 \cdot 1 \cdot 3^2}{1}$$

$$= \frac{1^2 + 1^2 + 2^2}{1} - 6$$

$$= \frac{1 + 1 + 4}{1} - 6$$

$$= \frac{6}{1} - 6$$

$$\frac{14 - 6}{6} = 2$$

Now $1 - 3 \cdot 1^2 = 6$
 $(14 + 6) - 18 = 6(1)^2$
 $-4^2 + 20 + 6 = 6^2 + 12 + 6$
 $10^2 - 8 = 0$
 $10 - 8 = 0$
 0

Q.22 Let $[t]$ denote the greatest integer less than or equal to t . Then the value of

$$\int_1^2 |2x - [3x]| dx \text{ is } \underline{\hspace{2cm}}$$

Sol. 1

$$\int_1^2 |2x - 3x| dx$$

$$3x = t$$

$$= \frac{1}{3} \int_3^6 \left| \frac{2t}{3} - [t] \right| dt$$

$$= \frac{1}{9} \int_3^6 |2t - 3[t]| dt$$

$$= \frac{1}{9} \left(\int_3^4 |2t - 9| dt + \int_4^5 |2t - 12| dt + \int_5^6 |2t - 15| dt \right)$$

$$= \frac{1}{9} \left(2t^2 - 9t \Big|_3^4 + 2t^2 - 12t \Big|_4^5 + 2t^2 - 15t \Big|_5^6 \right)$$

Q.25 For a positive integer n , $\frac{1}{x}^n$ is expanded in increasing powers of x . If three consecutive coefficients in this expansion are in the ratio, 2:5:12, then n is equal to _____

Sol. 118

Let 3 consecutive coH are

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} :: 2:5:12$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5} \quad \& \quad \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$$

$$\frac{r}{n-r+1} = \frac{2}{5} \quad \& \quad \frac{r+1}{n-r} = \frac{5}{12}$$

$$7r = 2n + 2 \quad \& \quad 17r = 5n - 12$$

$$\frac{2n + 2}{7} = \frac{5n - 12}{17}$$

$$= 34n + 34 = 35n - 84$$

$$n = 118$$