IIT-JEE (MAIN/ADVANCED) • NEET • BOARDS • NTSE • KVPY

MATHEMATICS QUESTION PAPER WITH SOLUTION

(CODE – PAPER 1)

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SECTION 1 (Maximum Marks : 18)

- This section contains **SIX** (06) questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +3	If ONLY the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −1	In all other cases.

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1. Suppose a,b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c,d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$ is

(A) 0 (B) 8000 (C) 8080 (D) 16000

Ans. **D**

2. If the function f:R R is defined by $f(x) = |x|(x-\sin x)$, then which of the following statements is TRUE ?

(A) f is one-one, but NOT onto (B) f is onto, but NOT one-one

(C) f is BOTH one-one and onto (D) f is NEITHER one-one NOR onto

Ans. **C**

3. Let the functions f:R R and g:R R be defined by

$f(x) = e^{x-1} - e^{-|x-1|}$ and $g(x) = \frac{1}{2}(e^{x-1}+e^{1-x})$.

Then the area of the region in the first quadrant bounded by the curves y = f(x), y = g(x) and x = 0 is.

(A) $(2\quad\sqrt{3})\quad\frac{1}{2}(e\quad e^{1})$ (B) $(2\quad\sqrt{3})\quad\frac{1}{2}(e\quad e^{1})$

(C) $(2\quad\sqrt{3})\quad\frac{1}{2}(e\quad e^{1})$ (D) $(2\quad\sqrt{3})\quad\frac{1}{2}(e\quad e^{1})$

Ans. **A**

4. Let a, b and be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\ x$, and suppose the ellipse $\frac{x^2}{a^2}\quad\frac{y^2}{b^2}\ 1$ passes through the point P. If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is.

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$

Ans. **A**

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5. Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose is the number of heads that appear when C_1 is tossed twice, independently, and suppose is the number of heads that appear when C_2 is tossed twice, independently. Then the probability that the roots of the quadratic polynomial $x^2 - x +$ are real and equal, is

(A) $\frac{40}{81}$ (B) $\frac{20}{81}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. **B**

6. Consider all rectangles lying in the region (x, y) R R : 0 x $\frac{}{2}$ and 0 y 2 sin(2x) and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is

(A) $\frac{3}{2}$ (B) (C) $\frac{}{2\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$

Ans. **C**

SECTION 2 (Maximum Marks : 24)

- This section contains **SIX** (06) questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four options(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correc answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –2	In all other cases.

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7. Let the function f:R R be defined by $f(x) = x^3 - x^2 + (x - 1) \sin x$ and let g:R R be an arbitrary function. Let fg: R R be the product function defined by (fg)(x) = f(x)g(x). Then which of the following statements is/are TRUE ?

(A) If g is continuous at x = 1, then fg is differentiable x = 1
(B) If fg is differentiable at x = 1, then g is continuous at x = 1
(C) If g is differentiable at x = 1, then fg is differentiable at x = 1
(D) If fg is differentiable at x =1, then g is differentiable at x = 1

Ans. **A,C**

8. Let M be a 3 × 3 invertible matrix with real entries and let I denote the 3 × 3 identity matrix. If M^{-1} = adj (adj M), then which of the following statements is/are ALWAYS TRUE ?

(A) M = I (B) det M =1 (C) M^2 = I (D) (adj M^2) = I

Ans. B,C,D

9. Let S be the set of all complex numbers z satisfying $|z^2+z+1|$ = 1. Then which of the following statements is/are TRUE ?

(A) $\left|z \quad \frac{1}{2}\right| \quad \frac{1}{2}$ for all z S

(B) |z| 2 for all z S

(C) $\left|z \quad \frac{1}{2}\right| \quad \frac{1}{2}$ for all z S

(D) The set S has exactly four elements

Ans. B,C

10. Let x, y and z be positive real numbers. Suppose x, y and z are the lengths of the sides of a triangle opposite to its angles X, Y and Z, respectively. If $\tan\frac{X}{2} \quad \tan\frac{Z}{2} \quad \frac{2y}{x \quad y \quad z}$,

then which of the following statements is/are TRUE ?

(A)2Y=X+Z (B)Y=X+Z (C) $\tan\frac{x}{2} \quad \frac{x}{y \quad z}$ (D) $x^2 + z^2 - y^2 = xz$

Ans. B,C

11 Let L_1 and L_2 be the following straight lines.

$$L_1 : \frac{x \quad 1}{1} \quad \frac{y}{1} \quad \frac{z \quad 1}{3} \text{ and } L_2 : \frac{x \quad 1}{3} \quad \frac{y}{1} \quad \frac{z \quad 1}{1}$$

Suppose the straight line $L : \frac{x}{l} \quad \frac{y \quad 1}{m} \quad \frac{z}{2}$

lies in the plane containing L_1 and L_2 , and passes through the point of intersection of L_1 and L_2.

If the line L bisects the acute angle between the lines L_1 and L_2 , then which of the following statements is/are TRUE?

(A)3 (B) $l \quad m \quad 2$ (C)1 (D) $l \quad m \quad 0$

Ans. A,B

12. Which of the following inequalities is/are TRUE?

(A) $\int_0^1 x \cos x \, dx \quad \frac{3}{8}$

(B) $\int_0^1 x \sin x \, dx \quad \frac{3}{10}$

(C) $\int_0^1 x^2 \cos x \, dx \quad \frac{1}{2}$

(D) $\int_0^1 x^2 \sin x \, dx \quad \frac{2}{9}$

Ans. A,B,D

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round -off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +4	If ONLY the correct numerical value is entered;
Zero Marks	: 0	In all other cases.

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13. Let m be the minimum possible value of $\log_3 \left(3^{y_1} \quad 3^{y_2} \quad 3^{y_3} \right)$, where y_1, y_2, y_3 are real numbers for which $y_1 \quad y_2 \quad y_3 \quad 9$. Let M be the maximum possible value of $\left(\log_3 x_1 \quad \log_3 x_2 \quad \log_3 x_3 \right)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 \quad x_2 \quad x_3 \quad 9$. Then the value of $\log_2 \left(m^3 \right) \quad \log_3 \left(M^2 \right)$ is_____

Ans. 8.00

14. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, \ldots$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 \quad b_1 \quad c$, then the number of all possible values of c, for which the equality

$2 \left(a_1 \quad a_2 \quad \ldots \quad a_n \right) \quad b_1 \quad b_2 \quad \ldots \quad b_n$ holds for some positive integer n, is_____

Ans. 1.00

15. Let $f:[0, 2]$ be the function defined by

$$f(x)\;\;(3\;\;\sin(2x))\sin x\;\;\frac{}{4}\;\;\sin 3x\;\;\frac{}{4}$$

If $\;,\;$ $[0, 2]$ are such that $\{x\;\;[0, 2]: f(x)\;\;0\}\;\;[\;,\;]$, then the value of is_____

Ans. 1.00

16. In a triangle PQR, let $a\;\;QR\;, b\;\;RP$ and $c\;\;PQ$. If $|a|\;\;3,\;\;|b|\;\;4$ and $\dfrac{a\;\;(c\;\;b)}{c\;\;(a\;\;b)}\;\;\dfrac{|a|}{|a|\;\;|b|}$,

then the value of $|a\,b|^{2}$ is_____

Ans. 108.00

17. For a polynomial $g(x)$ with real coefficients, let m_g denote the number of distinct real roots of $g(x)$. Suppose S is the set of polynomials with real coefficients defined by

$$S\;\;\;x^2\;\;1^2\;a_0\;\;a_1 x\;\;a_2 x^2\,a_3 x^3\;:a_0, a_1, a_2, a_3$$

For a polynomial f, let f and f denote its first and second order derivatives, respectively. Then the minimum possible value of $m_f\;\;m_f$, where $f\;\;S$, is_____

Ans. 5.00

18. Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit $\lim\limits_{x\;0}\dfrac{(1\;\;x)^{\frac{1}{x}}\,e^{\;1}}{x^a}$ is equal to a nonzero real number, is_____

Ans. 1.00